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Moment Coefficients With
Reference Position

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ABSTRACT

The variation of the aerodynamic force and moment coefficients
with change in the reference position are derived and given in both
the Ballistic and the Aerodynamic nomenclature. Also given is the
association between the Ballistic and Aerodynamic coefficients for
configurations having trigonal or greater rotational symmetry and
mirror symmetry.

INTRODUCTION

The variation of the aerodynamic forces and moments with change in the reference position on a missile or the variation of the aerodynamic forces and moments with center of gravity position on two missiles having the same configuration and flying at the same Mach number and Reynolds number, is given by Nielsen and Synge.[1] Hailperin[2] has put these results into Ballistic coefficient form. Kelley, McShane and Reno give the variation in Ballistic coefficient form in their forthcoming book.[4] This present note is concerned with deriving the relations using standard aerodynamic nomenclature and also with showing the association between the Ballistic coefficients and modified aerodynamic coefficients for a configuration possessing trigonal or greater rotational symmetry and mirror symmetry.

For the practical problem of determining the static and dynamic forces and moments which act on a missile from free flight observations, it is helpful and often necessary to know the variation of these forces and moments with change in reference positions. It is, for example, a standard procedure to launch two missiles with different center of gravity locations but of identical configuration at the same Mach number and Reynolds number. The data obtained from the firings when coupled with the relations to be rederived in this note yield aerodynamic forces and moments which may not be readily obtained from a single flight.

THEORY

Basic Relations

For the derivation consider the orthogonal axis system fixed in the missile and illustrated in Figure 1, where the x-axis is the principle axis of mass symmetry of a missile and the y and z axes are normal thereto and to themselves. The corresponding components of the linear velocity of the missile in space are given by u, v, and w, and the components of the total angular velocity of the missile in space are given by p, q, and r. The components of the total aerodynamic force acting on the missile are given by X, Y, and Z and the components of the total aerodynamic moment acting on the missile are given by L, M, and N.

The fundamental equations for the association between the linear and the angular velocity at two points on the missile's axis separated by distance ℓ, follow from the well known kinematics of a rigid body. The linear velocities for two points are related by

$$\begin{vmatrix} u \\ v \\ w \end{vmatrix}^{*} = \begin{vmatrix} u \\ v \\ w \end{vmatrix} + \begin{vmatrix} p \\ q \\ r \end{vmatrix} \times \begin{vmatrix} \ell \\ 0 \\ 0 \end{vmatrix} \tag{1}$$

Y, M
v, q

X, L
u, p

Z, N
w, r
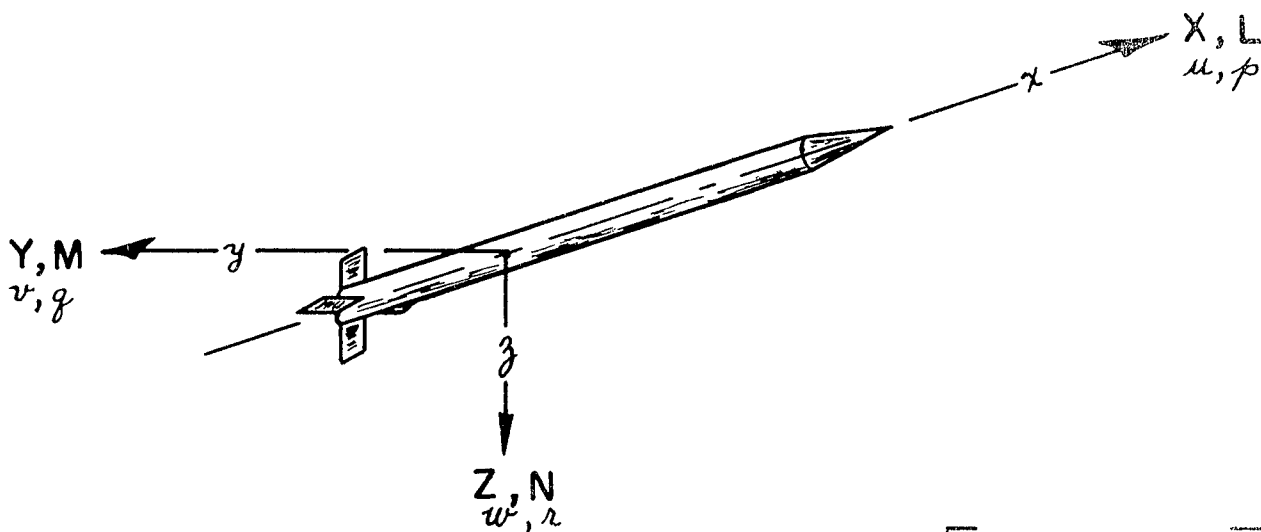
$S =$ REF. AREA $\left(\frac{\pi}{4} d^2\right)$

$d =$ BODY DIAMETER

$c =$ FIN CHORD

$b =$ FIN SPAN

$\ell =$ DISTANCE BETWEEN TWO REF. POINTS ON "x" AXIS

$$\text{FORCE} = \begin{bmatrix} X = C_X \frac{1}{2} \rho V^2 S \\ Y = C_Y \frac{1}{2} \rho V^2 S \\ Z = C_Z \frac{1}{2} \rho V^2 S \end{bmatrix}$$

$$\text{MOMENT} = \begin{bmatrix} L = C_\ell \frac{1}{2} \rho V^2 S b \\ M = C_m \frac{1}{2} \rho V^2 S c \\ N = C_n \frac{1}{2} \rho V^2 S b \end{bmatrix}$$

$$\text{LINEAR VEL} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = V$$

$$\text{ANGULAR VEL} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

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and the angular velocities for two points are related by

$$\begin{vmatrix} p \\ q \\ r \end{vmatrix}^* = \begin{vmatrix} p \\ q \\ r \end{vmatrix} \tag{2}$$

Since the total aerodynamic force acting on the missile[+] at a given instant is independent of the reference position selected on the missile, the relation between the forces acting at any two points is given by

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix}^* = \begin{vmatrix} X \\ Y \\ Z \end{vmatrix} \tag{3}$$

The total aerodynamic moment acting at any point is given by the moment at the known point plus the change in moment which is given by the cross multiplication of the force vector at the known point by the vector distance between the two points, namely

$$\begin{vmatrix} L \\ M \\ N \end{vmatrix}^* = \begin{vmatrix} L \\ M \\ N \end{vmatrix} + \begin{vmatrix} X \\ Y \\ Z \end{vmatrix} \times \begin{vmatrix} \ell \\ 0 \\ 0 \end{vmatrix} \tag{4}$$

It is convenient, at this point, to introduce complex quantities which combine the variables along the y and z axes. Accordingly, the pitching and yawing angular velocities are given by the quantity

$$\eta = q + ir \tag{5}$$

and the cross velocities, v and w, in terms of the angle of attack and the angle of yaw are given by

$$\zeta = \frac{v + iw}{u} = \beta + i\alpha \tag{6}$$

where $u \gg v, w$

From the above definitions, eq. (5) and eq. (6), and the basic relations, eq. (1) - eq. (4), the components along the missile axis and normal thereto are given by

$$\zeta^* = \zeta - i \frac{\ell}{u} \eta \tag{7}$$

$$\eta^* = \eta \tag{8}$$

[+]   For convenience the starred quantities may be considered to refer to any reference position on the x-axis and the unstarred quantities to refer to the center of gravity of the missile. The positive direction is forward.

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$$\left[Y + iZ\right]^* = \left[Y + iZ\right] \tag{9}$$

$$\left[M + iN\right]^* = \left[M + iN\right] - i\,\ell\left[Y + iZ\right] \tag{10}$$

$$u^* = u \tag{11}$$

$$p^* = p \tag{12}$$

$$X^* = X \tag{13}$$

$$L^* = L \tag{14}$$

Aerodynamic Force and Moment System

It is generally assumed [1,3,4] that the aerodynamic force and moment acting normal to the axis of mass symmetry (x-axis) on a symmetric missile[+] is linear in ζ and η . Accordingly the force and moment at each reference point are given by

$$Y + iZ = a\,\zeta + b\,\eta \qquad\qquad \left[Y + iZ\right]^* = a^*\zeta^* + b^*\eta^*$$

$$\tag{15}$$

$$M + iN = A\,\zeta + B\,\eta \qquad\qquad \left[M + iN\right]^* = A^*\zeta^* + B^*\eta^*$$

where $a,\ b,\ A,\ B,\ a^*,\ b^*,\ A^*$ and B^* are complex quantities.

Derivation of Basic Relations

First consider the force coefficients. Eqs. (15) are substituted into eq. (9) yielding

$$a^*\zeta^* + b^*\eta^* = a\,\zeta + b\,\eta \tag{16}$$

and substituting eq. (7) and eq. (8)

$$(a^*-a)\zeta + (b^*-ia^*\frac{\ell}{u} - b)\eta = 0 \tag{17}$$

from which follows

$$a^* = a \tag{18}$$

$$b^* = b + i\frac{\ell}{u}\,a \tag{19}$$

+ i.e. A missile possessing trigonal or greater rotational symmetry and mirror symmetry.

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and in terms of the real and imaginary parts[+]

$$a_1^* = a_1 \qquad (20)$$

$$a_2^* = a_2 \qquad (21)$$

$$b_1^* = b_1 - \frac{\ell}{u} a_2 \qquad (22)$$

$$b_2^* = b_2 + \frac{\ell}{u} a_1 \qquad (23)$$

The relations for the moment coefficients follow in a similar manner. Eqs. (15) are substituted into eq. (10) and then eq. (7) and eq. (8) are substituted yielding

$$(A^* - A + i\ell a)\, \xi + (B^* - i\frac{\ell}{u} A^* - B + i\ell b)\, \eta = 0 \qquad (24)$$

from which follows

$$A^* = A - i\,\ell a \qquad (25)$$

$$B^* = B - i\,\ell b + i\frac{\ell}{u} A + \frac{\ell^2}{u} a \qquad (26)$$

and in terms of the real and imaginary parts[+]

$$A_1^* = A_1 + \ell a_2 \qquad (27)$$

$$A_2^* = A_2 - \ell a_1 \qquad (28)$$

$$B_1^* = B_1 + \ell b_2 - \frac{\ell}{u} A_2 + \frac{\ell^2}{u} a_1 \qquad (29)$$

$$B_2^* = B_2 - \ell b_1 + \frac{\ell}{u} A_1 + \frac{\ell^2}{u} a_2 \qquad (30)$$

Variation of the Ballistic Coefficients With Reference Position

In the Ballistic nomenclature[++] the constants of eq. (15) are given by 3,4

[+] The subscript 1 refers to the real part and the subscript 2 refers to the imaginary part of the quantity.

[++] (Signs are obtained by considering that all the forces act ahead of the center of gravity.)

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$$a = -\rho d^2 u^2 K_M + i\rho d^3 \omega u K_F \qquad (31)$$

$$b = \rho d^4 \omega K_{XF} + i\rho d^3 u K_S \qquad (32)$$

$$A = -\rho d^4 \omega u K_T - i\rho d^3 u^2 K_M \qquad (33)$$

$$B = -\rho d^4 u K_H + i\rho d^5 \omega K_{XT} \qquad (34)$$

Now substituting eq. (31) - eq. (34) into eq. (27) - eq. (30) yields

$$K_M^* = K_M \qquad (35)$$

$$K_F^* = K_F \qquad (36)$$

$$K_S^* = K_S - \frac{\ell}{d} K_N \qquad (37)$$

$$K_{XF}^* = K_{XF} - \frac{\ell}{d} K_F \qquad (38)$$

$$K_M^* = K_M - \frac{\ell}{d} K_N \qquad (39)$$

$$K_T^* = K_T - \frac{\ell}{d} K_F \qquad (40)$$

$$K_H^* = K_H - \frac{\ell}{d} (K_S + K_M) + \frac{\ell^2}{d^2} K_N \qquad (41)$$

$$K_{XT}^* = K_{XT} - \frac{\ell}{d} (K_{XF} + K_T) + \frac{\ell^2}{d^2} K_F \qquad (42)$$

Variation of the Aerodynamic Coefficients with Reference Position

In an aerodynamic nomenclature the constants of eq. (15) are given by

$$a = C_{N_\beta} \frac{1}{2} \rho V^2 S + i C_{N_r} \left(\frac{pb}{2V}\right) \frac{1}{2} \rho V^2 S \qquad (43)$$

$$b = C_{N_r \eta}\left(\frac{pb}{2V}\right)\left(\frac{c}{2V}\right)\frac{1}{2}\ \rho V^2\ S\ \ + i\ C_{N_r \eta}\left(\frac{c}{2V}\right)\frac{1}{2}\ \rho V^2\ S \tag{44}$$

$$A = C_{N_{p}\zeta}\left(\frac{pb}{2V}\right)\frac{1}{2}\ \rho V^2\ S\ \ c\ - i\ C_{M\zeta}\ \frac{1}{2}\ \rho V^2\ S\ \ c \tag{45}$$

$$B = C_{M\eta}\left(\frac{c}{2V}\right)\frac{1}{2}\ \rho V^2\ S\ \ c\ - i\ C_{M_p \eta}\left(\frac{pb}{2V}\right)\left(\frac{c}{2V}\right)\frac{1}{2}\ \rho V^2\ S\ \ c \tag{46}$$

Now substituting eq. (43) - eq. (46) into eq. (27) - eq. (30) yields

$$C_{N_p \zeta}^* = C_{N\zeta} \tag{47}$$

$$C_{N_p \zeta}^* = C_{N_p \zeta} \tag{48}$$

$$C_{N\eta}^* = C_{N\eta} - \frac{2\ell}{c}\ C_{N\zeta} \tag{49}$$

$$C_{N_p \eta}^* = C_{N_p \eta} - \frac{2\ell}{c}\ C_{M_p \zeta} \tag{50}$$

$$C_{M\zeta}^* = C_{M\zeta} + \frac{\ell}{c}\ C_{N\zeta} \tag{51}$$

$$C_{M_p \zeta}^* = C_{M_p \zeta} + \frac{\ell}{c}\ C_{N_p \zeta} \tag{52}$$

$$C_{M\eta}^* = C_{M\eta} + \frac{\ell}{c}\ C_{N\eta} + \frac{2\ell}{c}\ C_{M\zeta} + \frac{2\ell^2}{c^2}\ C_{N\zeta} \tag{53}$$

$$C_{M_p \eta}^* = C_{M_p \eta} + \frac{\ell}{c}\ C_{N_p \eta} - \frac{2\ell}{c}\ C_{M_p \zeta} - \frac{2\ell^2}{c^2}\ C_{N_p \zeta} \tag{54}$$

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ASSOCIATION OF THE AERODYNAMIC COEFFICIENTS
AND THE BALLISTIC COEFFICIENTS

The association of the Ballistic coefficients and the Aerodynamic coefficients for missiles having trigonal or greater rotational symmetry and mirror symmetry follows from eq. (31) – eq. (34) and eq. (43) – eq. (46) and are given below.

$$(-)\ C_{N_\zeta} = -K_N\ 2\left(\frac{d^2}{S}\right) = C_{Z_\alpha} = C_{Y_\beta} \tag{55}$$

$$(+)\ C_{N_{p\zeta}} = +K_F\ 4\left(\frac{d^3}{bS}\right) = C_{Z_{p\beta}} = -C_{Y_{p\alpha}} \tag{56}$$

$$(-)\ C_{N_{p\eta}} = +K_{XF}\ 8\left(\frac{d^4}{bcS}\right) = C_{Z_{pr}}\left(\frac{b}{c}\right) = C_{Y_{pq}} \tag{57}$$

$$(-)\ C_{N_\eta} = +K_S\ 4\left(\frac{d^3}{cS}\right) = C_{Z_q} = -C_{Y_r}\left(\frac{b}{c}\right) \tag{58}$$

$$(+)\ C_{M_{p\zeta}} = -K_T\ 4\left(\frac{d^4}{bcS}\right) = C_{m_{p\beta}} = C_{n_{pa}}\left(\frac{b}{c}\right) \tag{59}$$

$$(-)\ C_{M_\zeta} = +K_M\ 2\left(\frac{d^3}{cS}\right) = C_{m_\alpha} = -C_{n_\beta}\left(\frac{b}{c}\right) \tag{60}$$

$$(-)\ C_{M_\eta} = -K_H\ 4\left(\frac{d^4}{c^2S}\right) = C_{m_q} = C_{n_r}\left(\frac{b^2}{c^2}\right) \tag{61}$$

$$(-)\ C_{M_{p\eta}} = -K_{XT}\ 8\left(\frac{d^5}{bc^2S}\right) = C_{m_{pr}}\left(\frac{b}{c}\right) = -C_{n_{pq}}\left(\frac{b^2}{c^2}\right) \tag{62}$$

The **signs in** the parentheses preceding the Aerodynamic coefficients are those which the coefficients have for the case when all the forces act behind the center of gravity of the missile.

The coefficients in the second column are standard ballistic nomenclature [2,3,4]. Those in the third and fourth columns are standard aerodynamic nomenclature (See Figure 1). Since there is not a one to one correspondence between the ballistic and aerodynamic nomenclature the suggested new nomenclature of the first column might be convenient.

John D. Nicolaides

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